Coupled FEM-MMP for Computational Electromagnetics

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The Multiple Multipole Program (MMP) is a boundary method for computing electromagnetic fields, which is well established in high frequency electromagnetics and computational optics due to its flexibility in terms of accuracy control, field excitation, and fast convergence. As any other boundary method MMP cannot efficiently solve nonlinear problems. The purpose of this paper is to show a novel numerical method for treating local nonlinear regions within a large linear MMP model. The main idea of this approach is to apply the well-known discretization scheme of the domain Finite Element Method (FEM) within the nonlinear region and to couple it over a special numerical interface with the linear MMP model surrounding it. The theoretical details of this MMP-FEM coupling and practical examples are presented in this paper.

***Index Terms*—Finite element method, domain method, multiple multipole program, boundary method, nonlinear materials.**

I. Introduction

Boundary methods of computational electromagnetics such as the Boundary Element Method (BEM), Method of Moments (MoM), Method of Auxiliary Sources (MAS), Multiple Multipole Program (MMP) etc. approximate the unknown electromagnetic field within a computational domain by using various field sources distributed either over boundaries between different materials (BEM and MoM) or over the domains themselves (MAS and MMP) [1]. The common feature of all boundary methods is that they discretize only boundaries and not domains thus reducing the dimensions of the problem by one. This is a considerable advantage over domain methods such as the Finite Difference Method (FDM) or Finite Element Method (FEM) [2].

The said advantage of boundary methods is very important in case of industrial applications dealing with large models of enormous geometrical complexity. Due to their theoretical background domain methods need a relatively large air-box that is demanding to construct and mesh for models with considerable size and complexity. Boundary methods do not require such an air-box at all, which is their second significant advantage.

The main disadvantage of boundary methods is their lack of capability to treat nonlinear domains. In order to overcome this difficulty different hybrid boundary-domain methods were suggested in the past, such as for example FEM-BEM coupling [3]. Using FEM for nonlinear regions and involving BEM for linear ones removes the need for air-box but introduces an additional problem, namely numerical evaluation of singular integrals over the FEM-BEM interface, which is a very time-consuming part.

In this paper a theory of the FEM-MMP coupling is presented that removes the need for air-box and does not introduce any singular integrals over the coupling boundary. The method is simple, general, and intuitively clear.

This main purpose and original scientific contribution of this paper is manifold: (a) to suggest a theoretical background for a novel FEM-MMP coupling, (b) to demonstrate the discretization of the field equations by it, and (c) to show the accuracy of the method by using several simple examples.

II. Numerical Method

To explain the basic idea of the method a very simple magnetostatic problem, presented in Figure 1, is defined. The problem consists of a line current (I), as the field source, placed outside of the computational domain and a ferromagnetic cylinder (Ω_2) surrounded by air (Ω_1). To describe the magnetic field in the computational domain the following field formulation by using the reduced magnetic scalar potential is used [2]:

$$\Omega_1: \quad \vec{H}_{1t} = \vec{H}_s + \vec{H}_1 = \vec{H}_s - \nabla \Phi_1 \qquad (1)$$

$$\Omega_2: \quad \vec{H}_2 = -\nabla \Phi_2 \qquad (2)$$

where H_{1t}, H_s, H_1 is the total, source and reaction magnetic field of the domain 1, respectively. H_2 is the reaction magnetic field of the domain 2 and Φ_1, Φ_2 are the reduced scalar magnetic potentials of the domain 1 and 2, respectively.

To take the nonlinearity of the ferromagnetic cylinder into account the domain Ω_2 is meshed and the unknown function Φ_2 over it is approximated by using the scalar FEM approach.

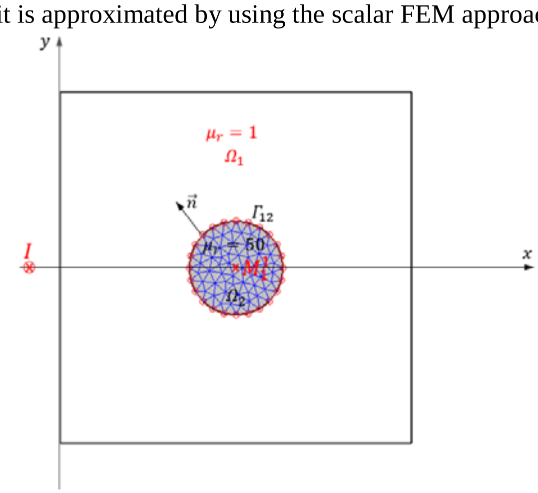


Fig. 1. A simple magnetostatic problem used to demonstrate the basic idea of the suggested FEM-MMP coupling is shown. The field source is a line current (I) outside of the computational domain. A ferromagnetic cylinder Ω_2 ($\mu_r=50$) is surrounded by air Ω_1 ($\mu_r=1$). More information can be found in the text.

The domain Ω_1 is linear and the unknown function Φ_1 over it is approximated by the MMP approach, i.e. by defining a multipole M_1^1 in the middle of the ferromagnetic cylinder.

The FEM and MMP approximations of the unknown function in the computational domain have the following form [1], [2]:

$$\Phi_1(r_1,\varphi_1) = B_0^1 \cdot \ln r_1 + \sum_{k=1}^{m} r_1^{-k} \left[B_k^1 \cdot \cos(k\varphi_1) + D_k^1 \cdot \sin(k\varphi_1) \right] \quad (3)$$

$$\Phi_2(x,y) = \sum_j N_j(x,y) \cdot \Phi_{2j} \quad (4)$$

Equation (3) represents a classical multipole expansion of *m-th* order, which is a standard MMP basis function. This is an analytical solution of the Laplace equation in the cylindrical coordinate system *(r_1, φ_1)* with the origin at the position of the multipole. Equation (4) is a classical FEM approximation of the scalar unknown function by using known shape functions N_j and unknown nodal values of the function Φ_{2j}.

Over the interface between two regions the following interface conditions must be fulfilled:

$$\Gamma_{12}: \quad \vec{n} \cdot \left(\mu_1 \vec{H}_{1t} \right) = \vec{n} \cdot \left(\mu_2 \vec{H}_2 \right) \quad (5)$$

$$\Gamma_{12}: \quad \vec{n} \times \vec{H}_{1t} = \vec{n} \times \vec{H}_2 \quad (6)$$

The coupling of the two methods can be explained by using the well-known FEM equivalent integral form of the magnetostatic boundary value problem (BVP) based on the reduced scalar magnetic potential:

$$\oint_{(\Gamma_{12})} \mu_1 N_i \, \vec{n} \cdot \vec{H}_1 \, dl + \iint_{(\Omega_2)} \mu_2 \nabla N_i \cdot \nabla \Phi_2 \, dS = - \oint_{(\Gamma_{12})} \mu_1 N_i \, \vec{n} \cdot \vec{H}_s \, dl \quad (7)$$

Equation (5) is already imposed in the boundary integral of Equation (7). It will be shown in the subsequent full paper that Equation (6) yields the following integral form:

$$\Phi_{2i} - \int_{P_i}^{P_N} \vec{t} \cdot \vec{H}_1 dt = \int_{P_i}^{P_N} \vec{t} \cdot \vec{H}_s dt \quad (8)$$

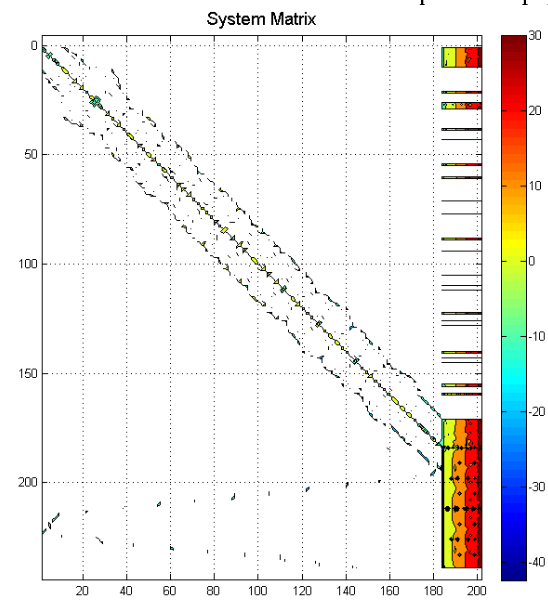
where Φ_{2i} is the value of the scalar magnetic potential at the *i-th* node on the boundary Γ_{12}, t is the tangential unit vector of the boundary Γ_{12}, P_N is the chosen reference point for zero value of the magnetic scalar potential (preferably on the boundary Γ_{12}), and P_i is the *i-th* node on the boundary Γ_{12}.

Equation (7) and (8) along with the FEM and MMP discretization of the unknown functions (5) and (6) yield the following overdetermined system of equations:

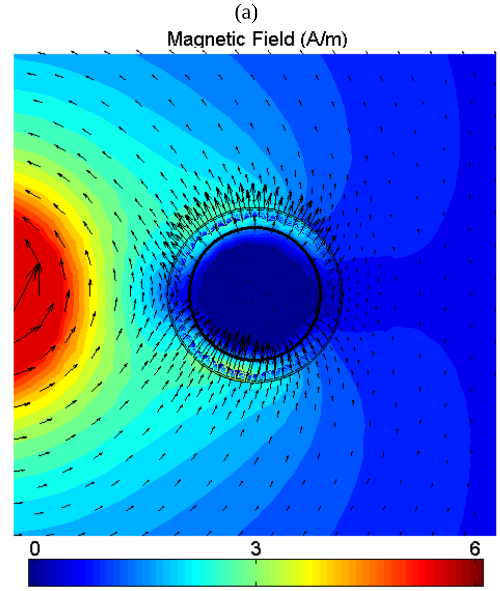
$$\begin{bmatrix} [F]_{N_n \times N_n} & [FM_{\odot}]_{N_n \times (2m+1)} \\ [MF_{\otimes}]_{N_{mp} \times N_n} & [M_{\otimes}]_{N_{mp} \times (2m+1)} \end{bmatrix} \begin{Bmatrix} \{\Phi_2\}_{N_n \times 1} \\ \begin{Bmatrix} B \\ D \end{Bmatrix}_{(2m+1)\times 1} \end{Bmatrix}_{[(N_n)+(2m+1)]\times 1} = \begin{Bmatrix} \{b\}_{N_n \times 1} \\ \{c\}_{N_{mp} \times 1} \end{Bmatrix} \quad (9)$$

where N_n is the number of nodes of the FEM mesh, N_{mp} is the number of matching points (boundary nodes of the FEM mesh), $\otimes$ denotes the matrix obtained by imposing the interface condition (6) and $\odot$ denotes the matrix obtained by imposing the interface condition (5). The unknowns of the system (9) are the coefficients of the function approximations (3) and (4). The matrix entries of the system (9) will be mathematically described in the subsequent full paper. The structure of the system matrix for the simple example presented in Figure 1 is shown in Figure 2a.

It is worth mentioning that the FEM-MMP interface works better if it is not defined exactly along the border of the ferromagnetic cylinder, but in the air near the cylinder as shown in Figure 2b (a thin air layer around the cylinder belongs also to the FEM part of the model). The numerical features of the method together with more demanding examples will be presented at the conference and in the subsequent full paper.



(a)



(b)

Fig. 2. FEM-MMP system matrix (9) in logarithmic scale (a) and magnetic field distribution (b) of the simple example defined in Fig 1. The radius of the ferromagnetic (μ_r=50) cylinder is *0.2m* and the position of the line current and cylinder are *(-0.01, 0)m* and *(0.075, 0)m*, respectively.

References

[1] C. Hafner, *Post-modern Electromagnetics*, John Wiley & Sons, Chichester, New York, London, 1999, pp. 217-277.

[2] J. M. Jin, *The Finite Element Method in Electromagnetics*, Third Edition, John Wiley & Sons, Chichester, New York, London, 2014, pp. 77-151.

[3] K. L. Wu, G. Y. Delisle, D. G. Fang, M. Lecours, *Coupled finite elements and boundary elements methods in electromagnetic scattering*, in *Finite Element and Finite Difference Methods in Electromagnetic Scattering*, M. A. Morgan, ed., Elsevier, New York, 1990, pp. 1-51.